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Analytical Solution of Black-Scholes Equation in Predicting Market Prices and Its Pricing Bias

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Authors' contributions

This work was carried out in collaboration between both authors. Author APA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author AIU managed the analysis of the study and literature searches. Two of us read and approved the final manuscript.

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Abstract

This paper is geared towards implementation of Black-Scholes equation in valuation of European call option and predicting market prices for option traders. First, we explained how Black-Scholes equation can be used to estimate option prices and then we also estimated the BS pricing bias from where market prices were predicted. From the results, it was discovered that Black-Scholes values were relatively close to market prices but a little increase in strike prices (K) decreases the option prices. Furthermore, goodness of fit test was done using Kolmogorov –Sminorvov to study BSM and Market prices.

Keywords: Black- Scholes Model (BSM); call option; bias and market prices.

1 Introduction

An option is a tool whose worth is derived from the principal asset which is otherwise known as financial derivative. This type of derivative does not have anything in common with mathematical meaning of

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derivative. The relevance of options valuation was first demonstrated by [1] when option faced difficulties in valuation of option at expiration. They used no-arbitrage argument to explain a partial differential equation which governs the growth of the option price with esteem to the expiration and cost of the fundamental Asset. The Black-Scholes equation has been used widely in many financial applications. So, in the theory of option pricing Black and Scholes will not be forgotten because of the remarkable effort made to option traders. Much research has been carried out to observe the pricing bias of Black-Scholes equation which was a great concern to many. Such as [2,3,4,5,6,7,8] etc.

In this paper, we shall be interested in Black-Scholes Model (BSM) for valuation of European Call option which have gained the interest of researchers and this interest is driven by demand of applications of societal problems. Secondly, estimating BS pricing bias and predicting of Market prices; goodness of fit test will be conducted to ascertain if option and market prices comes from a common distribution.

In financial markets, investors and financial analysts are generally too interested on how to maximize profit over particular trading days, that is, the changes in the price of goods and services. Therefore, modeling a behavior of a stock exchange market can be made through its relative change of the unstable market variables in time so as to predict stock price fluctuation, advice investors and corporative owners who are working out for convenient ways to do business by issuing of stocks in their corporations. Now, the market price behavior shows the characteristics as a stochastic process called “Brownian motion (BM)” or Wiener process with drift. It is an important example of stochastic processes satisfying a Stochastic Differential Equation (SDE) displayed in Fig. 1.

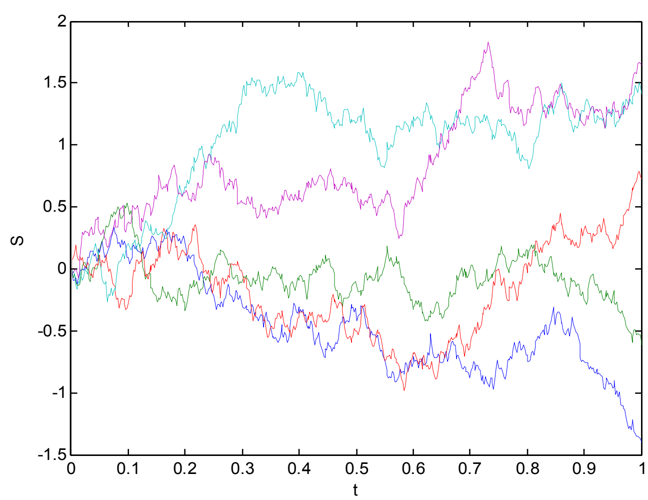

Fig. 1. Sample trajectories of the stock price process following Black-Scholes Model

Here, one of the characteristics of BM stipulates that the process is normally distributed with mean equal zero and hence negative with probability $\frac{1}{2}$ where the price of the stock normally grows at several rate, one way to handle this issues is to model stock price as a sum of a positive deterministic task of time and BM.

2 Mathematical Formulations

In the work of [1] the following seven assumptions were made:

- The asset price has properties of a Brownian motion with μ and σ as constants.
- There are no transaction costs or taxes.
- All securities are perfectly divisible.
- There is no dividend during the life of the derivatives.
- There are no riskless arbitrage opportunities.
- The security trading is continuous.
- The option is exercised at the time of maturity for both call and put options.

Definition 2.1

A call option a type of option that gives the owner the right to buy a single share of common stock.

Definition 2.2

An exercise price (strike price) is the amount paid for an asset when the option is exercised.

Definition 2.3

A European option is a kind of option that can be exercised only on a specified date in the future for fixed price.

According to [7,9,10,11] the derivation of BS PDE is based on Itô process with an assumption that the stock prices follow a geometric Brownian motion, ie

$$dS = \mu S_t dt + \sigma S_t dx \tag{1}$$

where S is the stock price, μ is the drift, σ is the volatility of underlying asset and dx is a Wiener process(Brownian Motion).

The BS PDE for European Call Option with value $C(S,t)$ is given in the following equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{2}$$

With initial and boundary conditions for Call Option:

$$\left.\begin{aligned} &C(0,t) = 0 \\ &C(s,t) = s\, when \rightarrow \infty \\ &C(s,\mathrm{T}) = \max(s-K,0) \end{aligned}\right\} \tag{3}$$

Theorem 1. The value of Vanilla European call is given by

$$C(\mathrm{S},t) = C(\mathrm{S}(t),\mathrm{K},T-t) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{4}$$

Where

$$N(d) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{d} e^{-\frac{1}{2}} s^2 ds \tag{5}$$

The cumulative distribution for the standard normal distribution

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad and \; d_2 = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \tag{6}$$

Applying theorem 1 using (3)-(6) solves Black-Scholes equation otherwise called analytic formula for the prices of European call option:

$$\left.\begin{array}{l} C = SN(d_1) - Ke^{-rt} N(d_2) \\ d_1 = \dfrac{\ln\left(\dfrac{S}{K}\right) + \left(r + \dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 = d_1 - \sigma\sqrt{T} \end{array}\right\} \tag{7}$$

here C is Price of a call option, S is price of underlying asset, K is the strike price,r is the riskless rate, T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.

In Table 1, column 3 were obtained by setting up r=0.03, k=0.25, sigma 0.8-1.2 with three initial stock prices such as 20.18, 37.04 and 50.42 using Black-scholes model of (1). The biases were estimated by subtracting each stock price from the corresponding option price. Then adding the errors in each partition of Table 1 and dividing by the corresponding option price. See column 4. In column 5, Market price was derived by adding BS pricing bias to stock prices in each of the four partitions. The 7th^{column} is the differences between market prices and option prices.

Generally, we observed that a little increase in the strike price decreases the option price. This will not go well to the option trader within the stipulated trading days, hence strike price simply means predetermined price an option holder wants to sell or buy. Secondly introduction of higher strike prices increases the pricing bias of Black-Scholes equation. However, the option and market prices are relatively comparable. This predicted Market prices, serves as a guide to option traders who are working assiduously in making sure profit is maximized.

3 Results and Discussion

3.1 Kolmogorov-smirnov goodness of fit test

H_0: Black-Scholes call and Market prices come from a common distribution.
H_1: They are not from the same distribution.

Results displayed in Table 1, in 6th column at α = 0.01 with three initial stock prices shows reject in all comparisons. We can conclude that both samples come from a common distribution.

Table 1. Summary results of the Black-Scholes Model, the pricing bias and market prices

Strike prices(K)	*r=0.03, T=1, Sigma=0.25*					
	Stock prices (S_0)	**BSM**	**Bias**	**Market prices**	**BSM/Mkt prices**	**Diff.Mkt/Stock prices**
0.8	20.18	19.4036	0.2404	20.4204	Reject	1.0168
0.9		19.3066	0.2513	20.4313	Reject	1.1247
1		19.2096	0.2526	20.4326	Reject	1.223
1.1		19.1125	0.2539	20.4339	Reject	1.3014
1.2		19.0155	0.2552	20.4352	Reject	1.4197
0.8	27.04	36.2636	0.1338	37.1738	Reject	0.9102
0.9		36.1666	0.1342	37.1742	Reject	1.09102
1		36.0696	0.1345	37.1745	Reject	1.1049
1.1		35.9725	0.1349	37.1749	Reject	1.2024
1.2		35.8755	0.1353	37.1753	Reject	1.2998
0.8	45.36	44.5836	0.1088	45.4688	Reject	0.8852
0.9		44.4866	0.1091	45.4691	Reject	0.9825
1		44.3896	0.1093	45.4693	Reject	1.0797
1.1		44.2925	0.1095	45.4695	Reject	1.177
1.2		44.1955	0.1098	45.4698	Reject	1.2743
0.8	50.42	49.6436	0.09774	50.5177	Reject	0.8741
0.9		49.5466	0.09793	50.5179	Reject	0.9713
1		49.4496	0.09812	50.5181	Reject	1.0695
1.1		49.3525	0.09832	50.5183	Reject	1.1658
1.2		49.2555	0.09851	50.5185	Reject	1.263

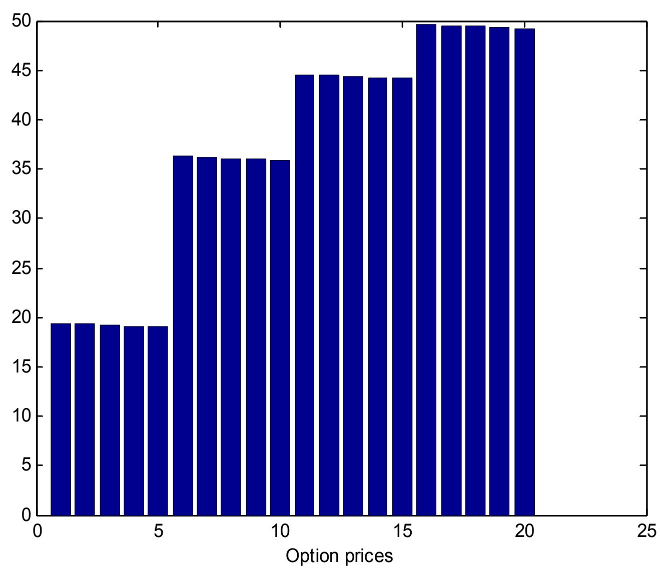

Fig. 1. Bar chart representation of market prices

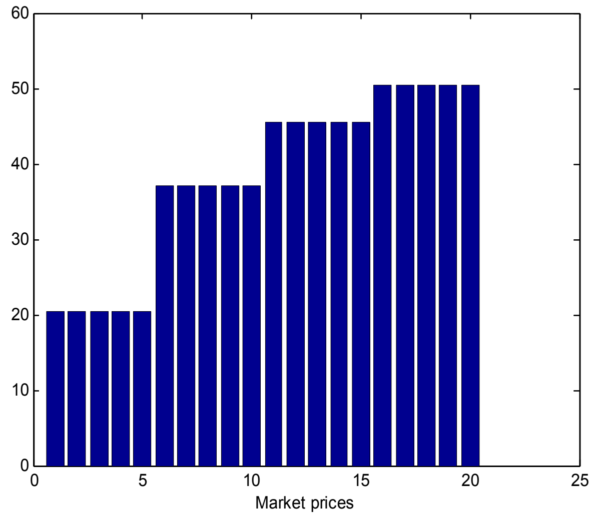

Fig. 2. Bar chart representation of predicted market prices

Figs. 1 and 2 are all significant, well correlated and shows that both belongs to a common distribution which is in agreement with our goodness of fit test. All these, is to show effectiveness of BS model.

4 Conclusions

This paper considered Black-Scholes equation via valuation of European call option prices; its bias and prediction of market prices. A brief analysis showed that introduction of higher strike prices increases the pricing bias of Black-Scholes equation. The method of estimating the bias from option prices before predicting market prices was done in other to give the option traders, decision makers and economist an sight of future market prices so as to plan ahead of time. However, Kolmogorov-Smirnov was used for goodness of fit test to identify a class of probability distributions, the random process which generated the two data – set (option and market prices). Howerer, a poor formulation of the BS model does not in any way mean that the model is unacceptable; there should be a continuous research, picking tools from Mathematics and Statistics to validate the Black and Scholes efforts of 1973.

Owing to the fact that, there's non-stationary in BS model, it is recommended to use controllability analysis to reflect the changes of stock prices in respect to time of expiration.

Finally, some consideration should be made before chosen strike prices because the aim of every business man is to maximize profit and minimize loss. Therefore, our novel contribution is unique, efficient and reliable in pricing European call option.

Competing Interests

Authors have declared that no competing interests exist.

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