

A Short Study on Bias Present in Classical Random Processes

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Short Research Article

Abstract

Sometimes, outcomes of random processes don't seem to follow the theoretical probabilities due to the presence of bias and even when the probabilities are followed in a large number of trials, dynamic bias is still evident in many of these processes. This paper provides a short study on the bias using examples and defines what kind of processes could be biased. It also demonstrates the two types of bias which are dynamic and fixed. This study could be used to analyze the bias in various random processes and get a better understanding of the outcomes. Dynamic bias has further been explained with the help of 52 cards. The study helps in providing a better understanding of randomness and further helps in designing experiments.

Keywords: Law of large numbers; monte carlo; bias; probability error; true probability; deck of 52 cards; design of experiments.

1 Introduction

A random experiment is an experiment that can be repeated multiple times under the same conditions and still give different outputs. In classical physics, no event can give a different outcome if repeated in the exact same manner [1,2]. For example, throwing a coin seems random until its velocity, angle of throw, height from the ground, and all the other variables are calculated and then, the outcome can be predicted with 100 percent accuracy. Because of this, the probability is believed to be the measure of ignorance. However, bias is often

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evident in the theoretically calculated probabilities, like in the game of darts, where the probability is calculated assuming that the odds for dart falling at a point is the same everywhere on the dartboard [3,4]. But, in the actual game, the probabilities might be different as the players aim at the center and the slice having more points [5]. A study by Diaconis et al. [6] also showed that flipping a coin could also be biased and for a natural coin flip, the chance of getting the face-up as started is 51 percent. Bias generally occurs due to hidden variables and misinterpretations made by humans. Even small misinterpretations can cause major changes in chaotic conditions.

The study provides the kinds of possible fundamental biases. These biases can be studied using the Monte Carlo method and then can be further divided fundamentally into two parts which are dynamic bias and fixed bias.

2 Methods

Bias with a significant effect on a large number of trials could be treated effectively using The Monte Carlo simulation.

Monte Carlo simulation is a mathematical method given by John von Neumann and Stanislaw Ulam, used to approximate the outcome of an event.

The idea is to simulate the experiment a large number of times and it will reveal the true probabilities of its outcomes. Considering the example of the game of darts again, the throwing of darts could be simulated with a large number of trials to achieve the probability distribution for the dartboard. After several hundred trials, the distribution will start to emerge (distribution can differ from person to person based on their skills).

2.1 Number of trials

The total number of trials for the Monte Carlo simulation should be large enough and may give different accuracy for the same number of trials on different experiments based on their total number of possible outcomes. Flipping a coin with only 2 outcomes might require fewer trials for the probability distribution to appear but experiments with a higher number of outcomes such as the game of roulette will require a greater number of trials to get the distribution with the same level of confidence. This can be understood using the equation:

$$T \propto O$$

Where T is the total number of trials and O is the total number of possible outcomes.

$$T = Oi$$

$$\frac{T}{O} = i \tag{1}$$

Here, i is the constant which signifies that for a particular value of i, the level of confidence is fixed for any random experiment.

3 Results and discussions

3.1 Dynamic bias

As the name suggests, Dynamic bias is not fixed to any particular outcome, it rather keeps on fluctuating. It occurs when the starting point (the way every element in the experiment is positioned) of a random experiment affects its outcome.

Axiom 1 - Dynamic bias is not evident in a large number of trials and the experiment always follows the law of large numbers.

Explanation:

The reason is because of its dynamic nature, all the outcomes are equally likely to get biased, and also this kind of bias is very small in most cases. For example, the coin flip is biased but still follows the law of large numbers. This bias could be turned into a fixed bias by starting with a particular face up every time flipping a coin. Monte Carlo simulation also fails to determine the dynamic bias as it has no significant effect on a large number of trials.

Conjecture 1- All the classical random processes where the orientation of the elements affects the probabilities of outcome are always dynamically biased.

Example:

Let's say we have a deck of 52 cards where we are trying to trace the location of the king of hearts after a single shuffle is done using the riffle technique. The deck is divided into two parts, group A with upper d cards and group B with lower $52-d$ cards, then the expected position of a card after a single shuffle whose current position is x in group A:

$$E(X) = \frac{52}{2d}(2x - 1) + .5 \tag{2}$$

Note: The above equation provides the mean value of the lower and upper bounds in which a particular card can exist if shuffled perfectly. For example- If d is 26 and x is 1, then after a perfect shuffle, the card can only exist in first or second place depending on the shuffler with the mean or expected value as 1.5.

After applying (2) to our king, the expected position for him can be known. The card shuffles are not perfect, hence the card of the king can be found outside its bound. The equation will give the expected value around which the probabilities are normally distributed for the location of the king after a single shuffle. However, if we are picking up the 30th card from the top after a single riffle shuffle as our outcome, assuming that deck is perfectly divided into two groups before the shuffle, the odds of getting the king of hearts is higher with 15 and 41 as the initial positions when compared to any other value. As the initial position of the king is affecting the probability of its outcome, this is an example of the dynamically biased random process. A paper by Bayer and Diaconis [7] stated that the bias decreases with a greater number of trials and a significant reduction occur after 7 shuffles.

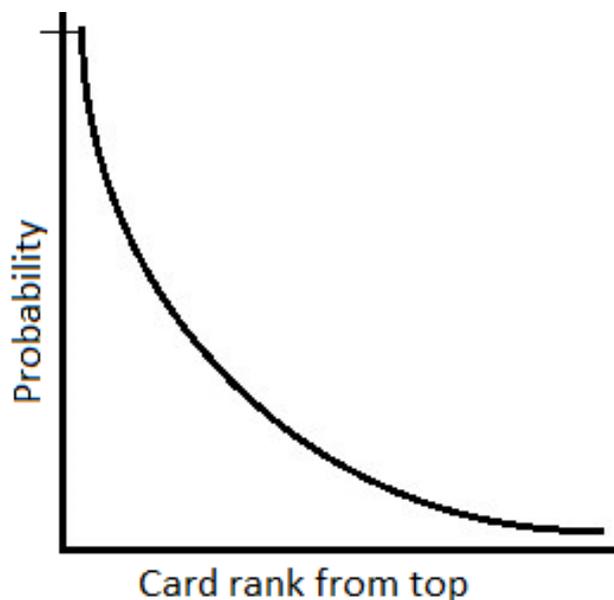


Fig. 1. Probability of getting a card in the same place as the initial position as we move further in Group A

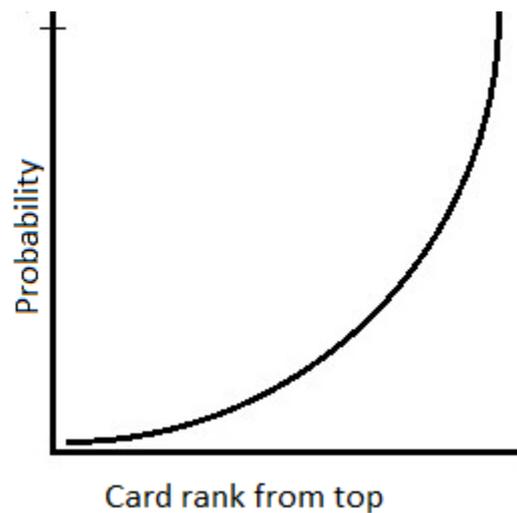


Fig. 2. Probability of getting a card in the same place as the initial position as we move further in Group B

The Fig. 1 and Fig. 2 can be derived from (2) and is valid for the division of deck in any ratio. In group A or the top part of the deck, the probability of getting a card in the same position after the shuffle decreases as we move ahead for cards with higher positions. The opposite of group A happens in group B, the probability trend increases with higher positions. So, it can be inferred from the Fig. 1 and Fig. 2 that if we are selecting the cards which have the same initial and final positions then, the top cards from group A and the last cards from group B will have higher odds of selection which again is an example of dynamic bias.

Explanation of Conjecture 1:

The dynamical bias arises because a certain position or positions get favored and the positioning of each element in the experiment defines the distribution of this bias. The experiment stays unaffected in long run as long as the distribution of elements stays random. Hence, whenever the orientation of the experiment causes any effect on the outcome, dynamical bias occurs.

3.2 Treating the dynamic bias

It is very hard to detect the dynamic bias as long as it stays dynamic, however, it can be turned to fixed bias and then can be analyzed easily. To convert dynamic bias to fixed, the experiment can be conducted by fixing the same orientation to each trial.

3.3 Fixed bias

Fixed bias is fixed to a particular outcome and if a random process is fixed biased, then the theoretical or biased probabilities aren't followed because the true distribution differs from the theoretical distribution under the Monte Carlo simulation.

Axiom 2- If the middle values of a symmetrical random distribution are negatively fixed biased, then the true probability distribution will tend to normal distribution.

Explanation:

Due to the presence of negative bias the true probabilities are underestimated. The increase in the probability of middle values will make the remaining curve area shrink as the total area of any probability distribution curve sums up to 1. Because of this behavior, the probability curve will tend to be more normal when compared to the theoretical or biased curve.

Assuming that the darts are thrown aiming at the center, then the game of darts is the perfect example of Axiom 2.

4 Conclusion

If simple processes such as throwing a dart or flipping a coin are biased, then it would not be wrong to conclude that other similar and complex processes can be biased as well. In fact, it is very common for a random process to be dynamically biased and most of the processes in classical physics are dynamically biased. However, even after using the Monte Carlo method with the same orientation, again and again, several cases with very low values of bias such as the coin are hard to determine. The study can be used in various fields such as gambling, design of experiments, random sampling, etc.

There still exists a problem regarding the kind of random processes which consists of probability fluctuation based on the orientation as described in Conjecture1. It is not clear that if all the classical random processes are dynamically biased in some way or not. Further study on this question will be required to get a better understanding of bias.

The work done by Diaconis et al. [7] was not at all easy in determining the bias of a simple coin flip, so it can be assumed that how tough it would be to determine the dynamical bias in complex random processes.

Competing Interests

Author has declared that no competing interests exist.

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