Abstract

Integer program was formulated in this work for efficient bus scheduling problem in Transport Company to obtain an optimal bus allocation system for its travelling routes in Nigeria. Integer programs were formulated for both the long and short routes categories. Transportation data from Akwa Ibom Transport Company (AKTC) was used to obtain an optimal bus schedule for the company with increase turnover in daily ticket collection by N2,327,500.00, N825,300.00 and N3,152,800.00 representing 34.78%, 10.60% and 21.78% respectively for the long routes, short routes and the overall routing system, while also meeting daily customers’ demand. Hence, the use of integer programming model is recommended for efficient vehicle scheduling and increase turnover in transport companies and personnel management system.

Keywords: Integer program; bus scheduling; branch and bound algorithm; transport company; travelling route.

1 Introduction

Operational planning within the public transit companies has been a major concern. Hence, the need to develop and refine existing models/methodologies for efficient transport system. According to [1], these models include maps, activity charts, balance sheets, PERT network, break-even equation, economic ordering, quantity
equation, scheduling and simulation, among others. Scheduling was described by [2] as a decision-making process that is used on a regular basis in many manufacturing and services industries, dealing with allocation of resources to tasks over given time period, with the goal of optimizing one or more objectives. While [3-6] defined scheduling as the temporal allocation of activities to resources to achieve some desirable objectives and categorize it into job process arrival, number of machines, the flow pattern and the performance criterion to be optimized.

A scheduling model based on interrelationship between passenger trip demands and bus trip supplies for inter-city carriers was formulated by [7] for efficient passengers’ services. Furthermore, [8] presented an efficient optimum solution for a real-life employee days-off scheduling problem to determine the minimum number of workers and feasible days-off assignments. Also, integer linear programming approach was applied by [8,9] and [10] to solve vehicle scheduling problem to cover all tasks and ensure that the number of drivers and costs of shifts is minimized. A real time scheduling method for a variable-route bus to reduce cost and average waiting time of the passengers was considered by [11]. In 2011, [12] underscored the use of buses in public transport technology as the most easily accessible and cheaper means compared to other kinds of public transportation medium. However, [13] showed that there is a declining reliability of bus services with increasing distance between a bus stop and the original terminal. Also, [14] proposed an integer linear programming model for exact solutions, considering skip-stop patterns when designing timetables for express trains and proposed several nonlinear integer programming models with linear constraints. However, [15] built an optimal model to schedule the departure time of each bus service with the objective of minimizing the average waiting time. The model when compared with the existing bus scheduling system helped to reduce the waiting time using experimental results. On this premise, this work will adapt an integer programming modelling approach with days-off as decision variables proposed by [15] to obtain efficient days-off bus scheduling roster. Real time data was obtained from Akwa Ibom Transport Company (AKTC), Uyo, Nigeria.

1.1 Problem definition

Akwa Ibom Transport Company (AKTC), Uyo is one of the major bus operators in Akwa Ibom State with large fleet of buses. The company currently owns 378 active buses with average of 14 passengers travelling on 22 different routes within Nigeria, majorly through the south-south, south-east, north central and south-western regions of the country. These routes are categorized as long or short routes. The routes to the north central, North West and South West are referred to as long routes while routes to South-South and South-East are short routes on an 18-hours (5.00 to 23.00) and 7 days-a-week schedule. A route is said to be long if its trip distance from Uyo terminus is at least 500km. On the other hand, a route is said to be short if its trip distance from Uyo terminus is less than 500km. Buses of long routes, at arrival at specified long route destination, take the next day off and resumes work on the second day after arrival. This implies that, a bus of a long route works 4 days in a week and takes 3 alternating days off. Buses of short routes operate the full 18-hours, only enjoying a 6-hours rest. Each of the company’s routes has an average daily passenger demand, daily bus allocation, per ticket price and the required bus allocation.

The company observed that, most times, reserve buses or buses of shorts routes are converted to long routes at emergency to meet the demands of long routes passengers. Thus, the company wishes to explore the lucrative ticket prices of the long routes as well as meet overall passengers’ demand. The management, in addition, also needs an allocation system on the remaining buses (resulting from the schedule system of the long route buses) to the short routes, taking into consideration, the per trip ticket collection for each short route trip subject to availability of at least 15 buses at the Uyo terminus for emergency conditions.

Hence, this work seeks to obtain an efficient bus scheduling model that would optimize fleet size and ticket collections for the various routes to ensure optimum profit and guarantees customers satisfaction taking into consideration the 3 alternating days off for the long route drivers and other limiting factors. This will avoid redundancy of buses, optimize man hour of drivers, enhance efficiency and increase productivity of the company.

1.2 Assumptions of the Study

1. Company buses are uniquely assigned to drivers and are not shared among company branches.
2. Company bus travelling on a long route is unavailable for schedule on the next work day: an off day is the day a bus is unavailable, hence, buses have 4 days-on and 3 days-off, irrespective of the alternating nature of this arrangement.
3. Travel time between two consecutive stations is assumed to be constant.
4. All buses have the same capacity of 14-seaters.

2 Methodology

2.1 The branch and bound algorithm for solving integer programming problems

2.1.1 Initialization

Consider the following integer programming problem (ILPP):

\[
\begin{align*}
\text{Maximze: } Z &= c_0x_1 + c_1x_2 + \ldots + c_qx_q \\
\text{subject to: } a_{11}x_1 + a_{12}x_2 + \ldots + a_{1q}x_q &= \tau_1 \\
& \vdots \quad \vdots \quad \vdots \quad \vdots \\
& a_{p1}x_1 + a_{p2}x_2 + \ldots + a_{pq}x_p &= \tau_p \\
x_j &\geq 0 \in \mathbb{N}^q \text{ for all } j = 1, 2, 3, \ldots, q \\
\end{align*}
\]

Eqns (1) - (3) can be expressed in matrix form as:

\[
\text{max } Z = G_{xq}^T x_q \\
\text{Subject to: } A_{pq}^T x_p = B_{pq} \\
x_j \geq 0 \in \mathbb{N}^q \text{ for some or all } j = 1, 2, 3, \ldots, q
\]

Where:

\[
G_{xq} = \begin{bmatrix} c_0 & c_1 & \ldots & c_q \end{bmatrix}
\]

\[
A_{pq} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1q} \\
& \ddots & \vdots \\
a_{p1} & a_{p2} & \ldots & a_{pq} \end{bmatrix}
\]

\[
B_{pq} = \begin{bmatrix} \tau_1 \\
& \ddots \\
& \ddots \\
& \tau_p \end{bmatrix}
\]

2.1.2 Branching step

Let \(x_k\) be one of the basic variables which does not have an integer value and also has the largest fractional value.

Branch (or partition) the LP problem into 2 new LP sub-problems based on integer values of \(x_k\) that are immediately above and below its non-integer value. It is partitioned by adding 2 mutually exclusive constraints to the original LP problem. The constraints are; \(x_k \leq \lfloor x_k \rfloor\) and \(x_k \geq \lceil x_k \rceil + 1\).
Here \([x_k]\) is the integer portion of the current fractional value of the variable, \(x_k\). This is done to exclude the non-integer value of \(X_k\). The two new LP problems will be:

Maximize: \(Z = \sum_{j=1}^{n} c_j x_j\)

subject to: \(\sum_{j=1}^{n} a_{ij} x_j = b_i\)
\(x_k \leq [x_k] + 1 ; X_j \geq 0\)

and: Maximize \(Z = \sum_{j=1}^{n} c_j x_j\)

subject to: \(\sum_{j=1}^{n} a_{ij} x_j = b_i\)
\(x_k \geq [x_k] + 1 ; X_j \geq 0\)

2.1.3 Bound step

Obtain the optimal solution of the sub-problems. Let the optimal value of the objective functions of the sub-problems be \(Z_2\) and \(Z_3\) respectively. The best integer solution value becomes the lower bound on the integer LP problem. Let the lower bound be denoted by \(Z_L\).

2.1.4 Fathoming step

i. If a sub-problem yields an infeasible solution, terminate the branch.

ii. If a sub-problem yields a feasible non-integer solution, return to step 2.

iii. If a sub-problem yields a feasible integer solution, examine the value of the objective function. If the value is equal to the upper bound, an optimal upper solution is reached. But if it is not equal to the upper bound but exceeds the lower bound, this value is used as the new upper bound, then return to step 2. But, if this value is less than lower bound, terminate the branch.

2.1.5 Termination

The procedure of branching and bounding continues until no further sub-problem remains to be examined. At this stage, the integer solution corresponding to the current lower bound is the optimal all-integer programming problem solution.

2.2 Integer linear programming problem formulation for the long routes

An optimal schedule for the long route buses will take into consideration assurance of these days off for each long-distance bus while meeting the daily bus requirement for all long routes. We let:

\(x_1 = \text{Number of buses on Monday, Wednesday and Friday}\)
\(x_2 = \text{Number of buses on Tuesday, Thursday and Saturday}\)
\(x_3 = \text{Number of buses on Tuesday and Thursday}\)
\(x_4 = \text{Number of buses on Tuesday, Thursday and Saturday}\)
\(x_5 = \text{Number of buses on Friday, Sunday and Tuesday}\)
\(x_6 = \text{Number of buses on Friday, Sunday and Tuesday}\)
\(x_7 = \text{Number of buses on Wednesday, Friday and Sunday}\)

The implication of the definitions of \(x_1\) to \(x_7\) is that, for instance, on Mondays, number of buses on duty will be the sum of \(x_1, x_3, x_5\) and \(x_7\), while on Tuesdays, number of buses on duty will be the sum of \(x_1, x_3, x_5\) and \(x_7\) and so on. Thus, the corresponding integer linear program is given as:

\[\text{Minimize } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7\]

subject to:

\[\text{Monday Constraint: } x_1 + x_3 + x_5 + x_7 \geq r_1\]
Where:

\[ r_j = \text{daily bus requirement, } j = 1, 2, 3, ..., 7 \]

This implies that \( r_1, r_2, \ldots, r_7 \) represent bus requirement for Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday respectively.

Therefore, a formal integer linear program is formulated as follows:

\[
\text{Minimize } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \tag{6}
\]

subject to:

\[
\begin{align*}
\sum_{i=1}^{7} x_i & \geq r_1 \\
\sum_{i=2}^{7} x_i & \geq r_2 \\
\sum_{i=3}^{7} x_i & \geq r_3 \\
\sum_{i=4}^{7} x_i & \geq r_4 \\
\sum_{i=5}^{7} x_i & \geq r_5 \\
\sum_{i=6}^{7} x_i & \geq r_6 \\
x_i & \geq 0 \quad \text{for } i = 1, 2, 3, ..., 7
\end{align*}
\]  

Eqns (6) to (8) can be expressed in matrix form as:

\[
\begin{align*}
\text{Minimize } Z &= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \\
\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \\
\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{bmatrix} \\
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\end{align*}
\]  

subject to:

\[
\begin{align*}
\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{bmatrix} & \geq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \\
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} & \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\end{align*}
\]
Table 1. Travelling routes, passenger demand, bus availability and bus requirement

<table>
<thead>
<tr>
<th>Travelling route</th>
<th>Average daily passenger demand</th>
<th>Daily bus availability</th>
<th>Daily bus requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Routes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uyo-Ojuelegba</td>
<td>340</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>Uyo-Ajah</td>
<td>47</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Uyo-Maza Maza</td>
<td>31</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Uyo-Okota</td>
<td>29</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Uyo-Jabi</td>
<td>153</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>Uyo-Nyanya</td>
<td>107</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Uyo-Ibadan</td>
<td>35</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Uyo-Jos</td>
<td>45</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Uyo-Kaduna</td>
<td>25</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Short Routes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uyo-Benin</td>
<td>46</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Uyo-Enugu</td>
<td>98</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Uyo-Owerri</td>
<td>158</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Uyo-Warri</td>
<td>95</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Uyo-Bayelsa</td>
<td>42</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Uyo-Asaba</td>
<td>76</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Uyo-Umuahia</td>
<td>471</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td>Uyo-Calabar</td>
<td>1,104</td>
<td>75</td>
<td>79</td>
</tr>
<tr>
<td>Uyo-Port Harcourt</td>
<td>1,188</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>Uyo-Awka</td>
<td>74</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Uyo-Abakiliki</td>
<td>72</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Uyo-Onitsha</td>
<td>70</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Uyo-Aba</td>
<td>641</td>
<td>40</td>
<td>46</td>
</tr>
</tbody>
</table>

Table 1 reveals that some long routes’ daily bus requirements are similar, implying that, for those that are similar, the solution will be same. Based on this premise, we establish 6 groups: A, B, C, D, E, F, respectively. A consists Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes. B consists Uyo-Ajah and Uyo-Jos routes. C, D, E, and F are the Uyo-Ojuelegba, Uyo-Jabi, Uyo-Nyanya and Uyo-Kaduna routes, respectively. Therefore, Integer Linear Program would be formulated and analyze for each of these 6 groups.

The integer linear problem for group A (G_a) is given as:

\[
\text{Minimize } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7
\]

subject to:

\[
\begin{align*}
x_1 + x_3 + x_5 + x_7 & \geq 3 \\
x_2 + x_4 + x_6 + x_7 & \geq 3 \\
x_1 + x_3 + x_5 + x_6 & \geq 3 \\
x_2 + x_4 + x_5 + x_7 & \geq 3 \\
x_1 + x_2 + x_4 + x_6 & \geq 3 \\
x_2 + x_3 + x_5 + x_7 & \geq 3 \\
x_i & \geq 0 \quad \text{for } i = 1, 2, 3, \ldots, 7
\end{align*}
\]

and can be expressed in matrix form as:

\[
\begin{bmatrix}
    x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7
\end{bmatrix}
\]

\[
\text{Minimize } Z = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \begin{bmatrix}
    x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7
\end{bmatrix}
\]
Similarly, integer linear programming formulation with daily buses requirement for the other routes are: $G_B = 4$, $G_C = 25$, $G_D = 11$, $G_E = 8$ and $G_F = 2$ with values of $r_i$ on the right-hand side of the constraints as 4, 25, 11, 8 and 2 respectively.

2.3 Problem formulation for short routes

Let $Y_p$ be the number of buses allocated to the $p$th short route. Therefore,

\begin{align*}
y_1 &= \text{the number of buses allocated to Benin route} \\
y_2 &= \text{the number of buses allocated to Enugu route} \\
y_3 &= \text{the number of buses allocated to Owerri route} \\
y_4 &= \text{the number of buses allocated to Warri route} \\
y_5 &= \text{the number of buses allocated to Bayelsa route} \\
y_6 &= \text{the number of buses allocated to Asaba route} \\
y_7 &= \text{the number of buses allocated to Umuahia route} \\
y_8 &= \text{the number of buses allocated to Calabar route} \\
y_9 &= \text{the number of buses allocated to Port Harcourt route} \\
y_{10} &= \text{the number of buses allocated to Awka routes} \\
y_{11} &= \text{the number of buses allocated to Abakiliki route} \\
y_{12} &= \text{the number of buses allocated to Onitsa route} \\
y_{13} &= \text{the number of buses allocated to Aba route}
\end{align*}

Let $T_p$ be the per bus ticket collection calculated as

\begin{equation}
T_p = \text{Per Ticket Price} \times \text{Bus Capacity}
\end{equation}

The Per Ticket Price is shown in Table 2 for the various short routes and bus capacity is given as 14 seater. Thus,

\begin{equation}
T_p = 14(\text{Per Ticket Price})
\end{equation}

The management’s objective for the short routes is to maximize profit (based on the 378 – 67 – 15 buses left in the company), taking into consideration the daily bus requirement. Therefore, the integer linear problem will be formulated as follows:

\begin{equation}
\text{Maximize } \sum_{p=1}^{13} T_p Y_p = 296
\end{equation}

subject to: $\sum_{p=1}^{13} Y_p = 296$
where:

\[
\begin{align*}
  y_1 &= 4 \\
  y_2 &\leq 7 \\
  y_3 &\leq 12 \\
  y_4 &\leq 7 \\
  y_5 &\leq 3 \\
  y_6 &\leq 6 \\
  y_7 &\leq 34 \\
  y_8 &\leq 79 \\
  y_9 &\leq 85 \\
  y_{10} &\leq 6 \\
  y_{11} &\leq 6 \\
  y_{12} &\leq 5 \\
  y_{13} &\leq 46 \\
  y_p &\geq 0 \epsilon \mathbb{N}
\end{align*}
\]  

This integer linear problem can be presented in matrix form as:

\[
\begin{align*}
  \text{Maximize } Z &= \begin{bmatrix} 672 & 448 & 322 & 532 & 448 & 532 & 280 & 210 & 322 & 427 & 560 & 455 & 154 \end{bmatrix} \begin{bmatrix} y_1 \\
  y_2 \\
  y_3 \\
  y_4 \\
  y_5 \\
  y_6 \\
  y_7 \\
  y_8 \\
  y_9 \\
  y_{10} \\
  y_{11} \\
  y_{12} \\
  y_{13} \end{bmatrix} \\
  \text{subject to:} \\
  \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = 305
\end{align*}
\]
In general, the optimal integer solution is reached when a feasible integer solution is achieved at a node that has an upper bound greater than or equal to the upper bound at any other ending node. For the purpose of this work, the TORA Optimization Software would be used for the analysis to ease computation, ensure efficiency and accuracy of results.

3 Analysis and Results

The various steps in obtaining optimal solution and the sub problems involved are depicted in a branch and bound diagram. Fig. 1 shows a branch and bound diagram for the long route’s bus scheduling problem in group A but with optimal solution at iteration 2. Similar network diagrams were also obtained for problems group B to F for the long routes problem with 1 iteration for group B, 3 iterations with optimal solution at iteration 2 for group C, 11 iterations with optimal solution at iteration 2 for group D, 1 iteration for group E and 7 iterations with optimal solution at iteration 2 for group F.

| Table 2. Summary of optimal solution of bus schedule for long routes in groups A to F |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) | \( x_7 \) | \( Z \) |
| A               | 0   | 0   | 0   | 0   | 3   | 3   | 0   | 6   |
| B               | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 7   |
| C               | 6   | 6   | 6   | 6   | 6   | 7   | 7   | 44  |
| D               | 2   | 2   | 2   | 2   | 2   | 2   | 5   | 20  |
| E               | 2   | 2   | 2   | 2   | 2   | 2   | 2   | 14  |
| F               | 0   | 0   | 0   | 0   | 2   | 2   | 0   | 4   |

In general, the optimal integer solution is reached when a feasible integer solution is achieved at a node that has an upper bound greater than or equal to the upper bound at any other ending node. For the purpose of this work, the TORA Optimization Software would be used for the analysis to ease computation, ensure efficiency and accuracy of results.

Table 2 summarizes the optimal solutions obtained for bus schedule of long routes in groups A to F. It shows the number of buses with respective shift patterns to be assigned to a particular group of routes. Specifically, a total of 6, 7, 44, 20, 14 and 4 buses are to be assigned to the routes in groups A, B, C, D, E and F, respectively. The values of \( x_1 \), … \( x_7 \) for the various groups: A, B, C, D, E and F are the number of buses on respective off days. However, the short routes problem converges at one iteration with optimal solutions in Fig. 2.
Fig. 1. An 11-iteration Branch and Bound diagram for long route bus scheduling problem in Group A

Fig. 2. A one step Branch and Bound diagram for the short routes bus scheduling problem
Table 3. Optimal allocation of buses to short routes in AKTC, Uyo

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>$y_4$</th>
<th>$y_5$</th>
<th>$y_6$</th>
<th>$y_7$</th>
<th>$y_8$</th>
<th>$y_9$</th>
<th>$y_{10}$</th>
<th>$y_{11}$</th>
<th>$y_{12}$</th>
<th>$y_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal allocation</td>
<td>4</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>3</td>
<td>6</td>
<td>34</td>
<td>79</td>
<td>85</td>
<td>6</td>
<td>5</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>Prior allocation</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>30</td>
<td>75</td>
<td>80</td>
<td>5</td>
<td>4</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Deficit</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3 shows the optimal allocation, prior allocation and the deficit allocation of buses to the various short routes. The results imply that 4, 7, 12, 7, 3, 6, 34, 79, 85, 6, 5 and 42 buses should be allocated to Uyo-Benin, Uyo-Enugu, Uyo-Owerri, Uyo-Warri, Uyo-Bayelsa, Uyo-Asaba, Uyo-Umuahia, Uyo-Calabar, Uyo-Port Harcourt, Uyo-Awka, Uyo-Abakiliki, Uyo-Onitsha and Uyo-Abia routes, respectively to amass respective profit of $N 67,200.00, $N 44,800.00, $N 64,400.00, $N 53,200.00, $N 42,700.00, $N 12,000.00, $N 44,000.00, $N 161,000.00, $N 42,700.00, $N 12,000.00, $N 30,800.00 representing 33.33%, 16.67%, 20%, 16.67%, 0%, 20%, 13.33%, 5.33%, 6.25%, 20%, 50%, 0% and 5% in Table 4. The results further show that the daily bus requirements for each short route have been met, exception of the Aba route, where the optimum allocation of 42 buses is less than the required 46 by 4. This situation, however, will have minimal impact on the overall profit of the company, as the Aba route has the least contribution to the profit from sales of ticket, also see Table 4.

Table 4. Routes and their respective daily allocations (prior and optimal), per ticket price and prior ticket collections

<table>
<thead>
<tr>
<th>S/N</th>
<th>Routes</th>
<th>Prior daily allocation (a₁)</th>
<th>Optimal daily allocation (a₂)</th>
<th>Per ticket price (tp) (₦)</th>
<th>Prior ticket collection (a₃ x tp x 14) (₦)</th>
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</thead>
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<td>3</td>
<td>Uyo-Ibadan</td>
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<td>7</td>
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<td>Uyo-Nyanya</td>
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Average revenue increase in naira and their corresponding percentage for both the long and short routes are obtained in Table 5.

<table>
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<tr>
<th>S/N</th>
<th>Routes</th>
<th>Optimal ticket collections (₦)</th>
<th>Increase (₦)</th>
<th>Average increase (₦)</th>
<th>Average percentage(%) increase</th>
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<td>Lower Limit (a1 x tp x 14) = (p2)</td>
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Table 6. Schedule pattern for long routes problem in group A

Table 6 is a sample bus schedule pattern for long routes in group A. Data in Table 1 show daily allocation of 2 buses each to members of G_3: Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes with daily requirement of 3 buses each as against an optimal allocation of 6 buses to each group member in order to satisfy the daily requirement in Table 2. Similarly, optimal allocation of buses was obtained for groups B to F as contained in Table 2.
The values of the decision variables in Table 2, for instance $x_i = 0; i = 2, 3, 4, 7$ imply that zero number of idle buses is associated with Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Off shift to Kaduna route. Rather, 2 buses should be assigned to the Uyo-Maza Maza, Uyo-Okota and Uyo-Ibadan routes. Also, $x_i = 0; j = 5$ and 6 implies that 3 idle buses are associated with Friday-Sunday-Tuesday and Thursday-Saturday-Monday off shifts, respectively and same should be assigned to these routes, and so on. This arrangement would lead to an average increase in daily ticket collection for these routes by ₦133,000.00 which represents a 50% increase as shown in Table 5.

Similarly, data in Table 1 show that 2 and 3 buses were respectively allocated to Uyo-Ajah and Uyo-Jos routes with daily requirements of 4 buses as against an optimal allocation of 7 buses required daily for each of these routes to satisfy the daily requirement in Table 2. The values of $x_i = 1; i = 1, 2, 3, 4, 5, 6, 7$ implies that one bus should be assigned to the Uyo-Ajah and Uyo-Jos routes on Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Off shift, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday and Wednesday-Friday-Sunday, respectively. This assignment would increase daily ticket collection by ₦266,000.00 and ₦147,000.00, representing 100% and 33.33% for the Uyo-Ajah and Uyo-Jos routes respectively as shown in Table 4. Also, the required allocation to Uyo-Quelegba route in $G_c$ is 25 buses daily but with actual allocation of 21 buses. This is against an optimal allocation of 44 buses required to satisfy the daily requirement as shown in Table 2. Also, the values of $x_i = 6; i = 1, 2, 3, 4, 5$ in Table 2 shows that 6 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Friday-Sunday-Tuesday off shift while $x_i = 7; j = 6$ and 7 shows that 7 buses should be assigned to Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift, respectively to the Quelegba route. The daily ticket collection for the Quelegba route with this optimal allocation would increase by ₦598,500.00, representing 21.43% increase as shown in Table 5.

Actual bus allocation for the Jabi route in $G_9$ is 9 in Table 1 with daily requirement need of 11 buses. However, the optimal allocation is 20 buses to be able to satisfy the daily requirement of 11 buses. The values of the decision variables: $x_i = 2; i = 1, 2, 3, 4, 5$ in Table 2 implies that 2 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Friday-Sunday-Tuesday off shift while $x_i = 5; j = 6$ and 7 shows that 5 buses should be assigned to Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift, respectively to the Jabi route. The daily ticket collection for this route under this optimal allocation would increase by ₦90,000.00, representing 38.89% increase as shown in Table 5.

In addition, actual bus allocation for the Nyanya route in $G_9$ is 6 buses with daily requirement of 8 buses as shown in Table 1, whereas an optimal allocation of 14 buses is required to satisfy the daily requirement. Also, the values of the decision variables: $x_i = 2; i = 1, 2, 3, 4, 5, 6, 7$ in Table 2 imply that 2 buses should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday, Friday-Sunday-Tuesday, Thursday-Saturday-Monday and Wednesday-Friday-Sunday off shift to Nyanya route. From Table 5, the daily ticket collection for this route would increase by ₦280,000.00, representing 33.33% increase under this optimal arrangement.

Finally for the Kaduna route in $G_9$ the actual allocation is 2 with daily requirement of 2 in Table 1. However, an optimal allocation of 4 buses is required to satisfy its daily requirement. The decision variables’ value: $x_i = 0; i = 1, 2, 3, 4, 7$ in Table 2 implies that no bus should be assigned to Tuesday-Thursday-Saturday, Monday-Wednesday-Friday, Sunday-Tuesday-Thursday, Saturday-Monday-Wednesday and Wednesday-Friday-Sunday off shift to Kaduna route. Rather, 2 buses should be assigned to this route for Friday-Sunday-Tuesday and Thursday-Saturday-Monday off shift. From Table 5, this arrangement will increase the ticket collection by ₦147,000.00 representing 50% increase. Therefore, this propose bus scheduling system will yield an overall daily increase in ticket collection by ₦3,152,800.00 which represents a 21.78% increase.

4 Conclusion

In this work, a bus scheduling problem was considered with a study of AKTC, Uyo. The problem was formulated as integer linear program with branch and bound network diagrams for long routes in group A as shown in Fig. 1 and short routes as shown in Fig. 2. The optimal solutions are presented in Tables 2, 3 and 5.
These results show that for long routes, an optimal allocation of 6, 7, 44, 20, 14 and 4 buses should be made to the routes in groups A, B, C, D, E and F. For the short routes, it was shown that an optimal allocation of 4, 7, 12, 7, 3, 6, 34, 79, 85, 6, 6, 5 and 42 buses should be made to Uyo-Benin, Uyo-Enugu, Uyo-Owerri, Uyo-Warri, Uyo-Bayelsa, Uyo-Asaba, Uyo-Umuahia, Uyo-Calabar, Uyo-Port Harcourt, Uyo-Awka, Abakiliki, Uyo-Onitsha and Uyo-Abia travelling routes, respectively to maximize profit from ticket collections. These optimal solution would result in increased ticket sales on the long, short and overall routing by ₦2,327,500.00, ₦825,300.00 and ₦3,152,800.00 with corresponding percentages: 34.78%, 10.60% and 21.78% respectively, while meeting daily passenger demands for the various routes and ensuring proper bus allocation system. These results are in line with [6,9,10] and [16,17], among others who have shown that the use of integer program for vehicle, personnel and other scheduling problems yield efficient schedule, increase profit and overall turnover. Hence, the use of integer program (IP) model is recommended for scheduling purposes in transport companies, hotels, security organizations and power distributing companies, among others. This would ensure optimal allocation of resources for increase turnover and optimum profit.

Disclaimer

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

Competing Interests

Authors have declared that no competing interests exist.

References


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